CENG114 Lecture Notes

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1 Review Session 1

1.1 Example 1

$$A = \{a, e, i\}$$

$$A^C = \{b, c, d, f, g, h, j\}$$

$$B = \{c, d, e, f\}$$

$$B^C = \{a, b, g, h, i, j\}$$

Number of elements in $\Omega = 10$

i.

$$A \cap B = \{e\}, P(A \cap B) = \frac{1}{10}$$

ii.

$$A^C \cup B = \{b, c, d, e, f, g, h, j\}, P(A^C \cup B) = \frac{8}{10} = \frac{4}{5}$$

ii.

$$A^C \cap B^C = \{b, g, h, j\}, P(A^C \cap B^C) = \frac{4}{10} = \frac{2}{5}$$

1.2 Example 2

Let X, Y and Z denote a positive, inconclusive and negative test result respectively. Let A denote the event that there is high lead content.

$$
\begin{aligned}
P(X|A) &= 0.95 \\
P(Z|A) &= 0.02 \\
P(Y|A) &= 1 - 0.95 - 0.02 = 0.03 \\
P(Z|A^C) &= 0.97 \\
P(X|A^C) &= 0.01 \\
P(Y|A^C) &= 1 - 0.97 - 0.01 = 0.02 \\
P(A) &= 0.02
\end{aligned}
$$

a.

$$
\begin{aligned}
P(A|X) &= \frac{P(X|A)P(A)}{P(X)} \\
P(X) &= P(X|A)P(A) + P(X|A^C)P(A^C) \\
&= 0.95 \times 0.02 + 0.01 \times (1 - 0.02) \\
&= 0.0288 \\
P(A|X) &= \frac{0.95 \times 0.02}{0.0288} \\
&= 0.660
\end{aligned}
$$

b.

$$
\begin{aligned}
P(A^C|Z) &= \frac{P(Z|A^C)P(A^C)}{P(Z)} \\
P(Z) &= P(Z|A)P(A) + P(Z|A^C)P(A^C) \\
&= 0.02 \times 0.02 + 0.97 \times (1 - 0.02) \\
&= 0.951 \\
P(A^C|Z) &= \frac{0.97 \times (1 - 0.02)}{0.951} \\
&= 0.9996
\end{aligned}
$$

1.3 Example 3

$$\begin{aligned}
P(a \to c) &= 1 - (1 - 0.9)(1 - 0.9)(1 - 0.9) \\
&= 0.999 \\
P(c \to d) &= 1 - (1 - 0.95)(1 - 0.95) \\
&= 0.9975 \\
P(a \to b) &= 0.999 \times 0.9975 \times 0.99 \\
&= 0.9865
\end{aligned}$$

1.4 Example 4

i. Number of phone numbers = $390 \times 9 \times 10^6 = 3.51$ billion.

ii. Number of phone numbers = $3 \times \frac{7!}{1!2!4!} = 315$

iii. Number of phone numbers = $3 \times (3^7 - 3) = 6552$

1.5 Example 5

a.

Number of one-stage = 4

Number of two-stage = $4 \times 3 = 12$

Number of three-stage = $4 \times 3 \times 3 = 36$

Total number of possible experiments = $36 + 12 + 4 = 52$

b. p = $\frac{36}{52} = 0.692$