CENG114 Lecture Notes

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Contents

1 Lecture 10

1.1 Slide 10

$$\bar{X} = \frac{105 + 97 + 245 + 163 + 207 + 134 + 218 + 199 + 160 + 196}{10} = 172.4$$

i. 95% confidence interval $= 172.4 \pm 1.96\frac{34}{\sqrt{10}} = 151 - 193$

ii. 99% confidence interval $= 172.4 \pm 2.58\frac{34}{\sqrt{10}} = 144 - 200$

1.2 Slide 15

$$\begin{aligned} \bar{X} &= 530 \\ \sigma_{\bar{X}} &= \frac{\sigma}{\sqrt{30}} = \frac{30}{\sqrt{30}} = 5.477 \\ z &= \frac{\bar{X} - \mu_{hyp}}{\sigma_{\bar{X}}} = \frac{530 - 513}{5.477} = 3.103 \end{aligned}$$

1.3 Slide 19

$$\begin{aligned} \sigma_{\bar{X}} &= \frac{\sigma}{\sqrt{30}} = \frac{60}{\sqrt{30}} = 10.95 \\ z &= \frac{\bar{X} - \mu_{hyp}}{\sigma_{\bar{X}}} = \frac{530 - 513}{10.95} = 1.55 < 1.96 \end{aligned}$$

Therefore, we reject H_0.

1.4 Slide 21

$$z = = \frac{530 - 513}{10.95} = 1.55 < 1.645$$

Therefore, we do not reject H_0.

1.5 Slide 22

$$z = = \frac{530 - 513}{10.95} = 1.55 > -1.645$$

Therefore, we do not reject H_0.

1.6 Slide 30

i.

$$
\begin{aligned}
H_0 : &\qquad \mu = 162 \\
H_1 : &\qquad \mu \neq 162 \\
z = &\quad \frac{172-162}{\frac{34}{\sqrt{10}}} = 0.93 < z_{0.05} = 1.96
\end{aligned}
$$

Do not reject H_0 at $\alpha = 0.05$.

ii. P(type I error) $= \alpha = 0.05$.

iii. For a type II error to occur, a sample mean must result in $z > -z_{0.05}$.

$$
\begin{aligned}
\frac{\bar{X} - 162}{\frac{34}{\sqrt{10}}} &> -1.96 \\
\bar{X} &> 140.9 \\
P(\text{Type II error}) &= P(\bar{X} > 140.9) \\
&= P(Z > \frac{140.9 - 110}{\frac{34}{\sqrt{10}}}) \\
&= P(Z > 2.874) \\
&= 1 - P(Z < 2.874) \\
&= 0.002
\end{aligned}
$$

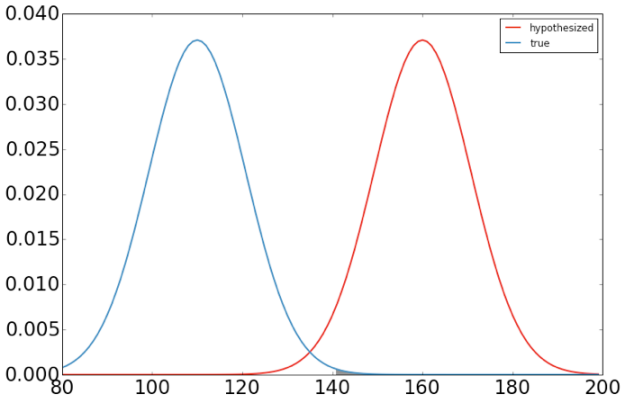

Figure 1: Type II error

1.7 Slide 39

$$
\begin{aligned}
H_0 &: \mu = 162 \\
H_1 &: \mu \neq 162 \\
\sum X &= 105 + 221 + 182 + 186 + 121 + 181 + 180 + 143 = 1319 \\
\sum X^2 &= 105^2 + 221^2 + 182^2 + 186^2 + 121^2 + 181^2 + 180^2 + 143^2 = 227837 \\
\bar{X} &= \frac{1319}{8} = 164.875 \\
SS_X &= \sum X^2 - \frac{(\sum X)^2}{8} = 227837 - \frac{1319^2}{8} = 10366.875 \\
s &= \sqrt{\frac{SS_X}{n-1}} = \sqrt{\frac{10366.875}{8-1}} = 38.48 \\
s_{\bar{X}} &= \frac{s}{\sqrt{n}} = \frac{38.48}{\sqrt{8}} = 13.60 \\
t &= \frac{\bar{X} - 162}{s_{\bar{X}}} = \frac{164.875 - 162}{13.6} = 0.211 \\
dof &= 8 - 1 = 7
\end{aligned}
$$

From the t-tables, we have $t_{crit} = \pm 2.365$. Since $t < t_{crit}$, we do not reject null hypothesis.

1.8 Slide 43

$\bar{X} \pm t_{conf} s_{\bar{X}} = 164.875 \pm 3.499 \times 13.6 = 117.3 - 212.5$

1.9 Slide 45

$$
\begin{aligned}
H_0 &: p \leq 0.1 \\
H_1 &: p > 0.1 \\
\hat{p} &\sim N(0.1, 0.00018) \\
z &= \frac{0.125 - 0.1}{\sqrt{0.00018}} = 1.863 > z_{crit} = 1.645
\end{aligned}
$$

Reject null hypothesis at 5% level.

1.10 Slide 52

$$
\begin{aligned}
H_0 &: \mu_1 - \mu_2 = 0 \\
H_1 &: \mu_1 - \mu_2 \neq 0 \\
\sum X_1 &= 105 + 221 + 182 + 186 + 121 + 181 + 180 + 143 = 1319 \\
\sum X_1^2 &= 105^2 + 221^2 + 182^2 + 186^2 + 121^2 + 181^2 + 180^2 + 143^2 = 227837 \\
\bar{X_1} &= \frac{1319}{8} = 164.875 \\
SS_{X_1} &= \sum X_1^2 - \frac{(\sum X_1)^2}{8} = 227837 - \frac{1319^2}{8} = 10366.875 \\
\sum X_2 &= 97 + 154 + 153 + 174 + 120 + 168 + 167 + 141 = 1174 \\
\sum X_2^2 &= 97^2 + 154^2 + 153^2 + 174^2 + 120^2 + 168^2 + 167^2 + 141^2 = 117204 \\
\bar{X_2} &= \frac{1174}{8} = 146.75 \\
SS_{X_2} &= \sum X_2^2 - \frac{(\sum X_2)^2}{8} = 117204 - \frac{1174^2}{8} = 4919.5 \\
s_P^2 &= \frac{SS_{X_1} + SS_{X_2}}{n_1 + n_2 - 2} = \frac{10366.875 + 4919.5}{8 + 8 - 2} = 1091.88 \\
s_{\bar{X_1} - \bar{X_2}} &= \sqrt{\frac{s_P^2}{n_1} + \frac{s_P^2}{n_2}} = \sqrt{\frac{2 \times 1091.88}{8}} = 16.52 \\
t &= \frac{164.875 - 146.75 - 0}{16.52} = 1.097 \\
dof &= 8 + 8 - 2 = 14
\end{aligned}
$$

From the t-tables, we have $t_{crit} = \pm 2.145$. Since $t < t_{crit}$, we do not reject null hypothesis.

1.11 Slide 54

$\bar{X_1} - \bar{X_2} \pm t_{conf} s_{\bar{X_1} - \bar{X_2}} = 164.875 - 146.75 \pm 2.977 \times 16.52 = -31.1 - 67.3$

1.12 Slide 60

$$
\begin{aligned}
H_0 &: \mu_D \geq 0 \\
H_1 &: \mu_D < 0 \\
SS_D &= \sum D^2 - \frac{(\sum D)^2}{8} = 5940 - \frac{(-146)^2}{8} = 3275.5 \\
s_D &= \sqrt{\frac{SS_D}{n-1}} = \sqrt{\frac{3275.5}{8-1}} = 21.6 \\
s_{\bar{D}} &= \frac{s_D^2}{\sqrt{n}} = \frac{21.6}{\sqrt{8}} = 7.648 \\
t &= \frac{-146/8 - 0}{7.648} = -2.386 \\
dof &= 8 - 1 = 7
\end{aligned}
$$

From the t-tables, we have $t_{crit} = -1.895$. Since $t < t_{crit}$, we reject null hypothesis.

1.13 Slide 61

$\bar{D} \pm t_{conf} s_{\bar{D}} = -\frac{146}{8} \pm 2.365 \times 7.648 = -0.162 - -2.412$

1.14 Slide 64

$$
\begin{aligned}
H_0 &: \rho = 0 \\
H_1 &: \rho \neq 0 \\
t &= \frac{r - \rho}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{0.951 - 0}{\sqrt{\frac{1-0.951^2}{7-2}}} = 6.878 \\
dof &= 7 - 2 = 5
\end{aligned}
$$

From the t-tables, we have $t_{crit} = \pm 6.869$. Since $t > t_{crit}$, we reject null hypothesis.