

CENG114 Lecture Notes

Shyue Ping Ong

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1 Lecture 04

1.1 Slide 7

1.

$$P(Y = y) = \begin{cases} \frac{1}{4} & \text{for } 1 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

2. Let us consider the results of the two dice separately as Y_1 and Y_2 and denote a result as (Y_1, Y_2) .

Number of permutations of $Y_1 Y_2 = 4 \times 4 = 16$.

To get a $X = 2$, we have $(1, 1)$.

To get a $X = 3$, we have $(1, 2)$ or $(2, 1)$.

To get a $X = 4$, we have $(2, 2)$, $(1, 3)$, $(3, 1)$.

To get a $X = 5$, we have $(2, 3)$, $(3, 2)$, $(1, 4)$, $(4, 1)$.

To get a $X = 6$, we have $(3, 3)$, $(2, 4)$, $(4, 2)$.

To get a $X = 7$, we have $(3, 4)$, $(4, 3)$.

To get a $X = 8$, we have $(4, 4)$.

$$p_X(x) = \begin{cases} \frac{1}{16} & \text{for } x = 2, 8 \\ \frac{1}{8} & \text{for } x = 3, 7 \\ \frac{3}{16} & \text{for } x = 4, 6 \\ \frac{1}{4} & \text{for } x = 5 \\ 0 & \text{otherwise} \end{cases}$$

1.2 Slide 8

Normalization:

$$\sum_{i=1}^8 p_X(x) = 2 \times \frac{1}{16} + 2 \times \frac{1}{8} + 2 \times \frac{3}{16} + \frac{1}{4}$$

If $S = \{x | x \text{ is even}\}$,

$$P(X \in S) = 2 \times \frac{1}{16} + 2 \times \frac{3}{16} = \frac{1}{2}$$

1.3 Slide 9

What is the value of $F_X(x)$ for $x < 2$?

$$F_X(x) = \begin{cases} 0 & x < 2 \\ \frac{1}{16} & x = 2 \\ \frac{3}{16} & x = 3 \\ \frac{3}{8} & x = 4 \\ \frac{5}{8} & x = 5 \\ \frac{13}{16} & x = 6 \\ \frac{15}{16} & x = 7 \\ 1 & x \geq 8 \end{cases}$$

Draw plot.

1.4 Slide 11

What are the possible values of Y ?

$$Y = 0, 1, 4, 9$$

$$P(Y = 0) = P(X = 0) = \frac{1}{7}$$

$$P(Y = 1) = P(X = -1) + P(X = 1) = \frac{2}{7}$$

$$P(Y = 4) = P(X = -2) + P(X = 2) = \frac{2}{7}$$

$$P(Y = 9) = P(X = -3) + P(X = 3) = \frac{2}{7}$$

$$p_Y(y) = \begin{cases} \frac{1}{7} & \text{for } y = 0 \\ \frac{2}{7} & \text{for } y = 1, 4, 9 \\ 0 & \text{otherwise} \end{cases}$$

1.5 Slide 15

$$E[X] = \frac{1}{2}$$

$$E[X] = 3.5$$

1.6 Slide 16

$$E[X] = \frac{1}{2} \times 0 + \frac{1}{2} \times 1 = 0.5$$

1.7 Slide 17

$$E[X] = \sum_{i=1}^8 xp_X(x) = \frac{1}{16}(2+8) + \frac{1}{8}(3+7) + \frac{3}{16}(4+6) + \frac{1}{4}(5) = 5$$

1.8 Slide 18

$$E[g(x)] = \sum_{i=1}^8 e^x p_X(x) = \frac{1}{16}(e^2 + e^8) + \frac{1}{8}(e^3 + e^7) + \frac{3}{16}(e^4 + e^6) + \frac{1}{4}e^5 = 449.34$$

1.9 Slide 22

$$E[X^2] = \sum_{i=1}^8 x^2 p_X(x) = \frac{1}{16}(2^2 + 8^2) + \frac{1}{8}(3^2 + 7^2) + \frac{3}{16}(4^2 + 6^2) + \frac{1}{4}(5^2) = 27.5$$

$$\text{var}(X) = E[X^2] - E[X]^2 = 27.5 - 5^2 = 2.5$$

$$\sigma_X = \sqrt{\text{var}(X)} = 1.581$$

1.10 Slide 30

$$\text{Area under curve} = c^2 + \frac{1}{2}c^2 = \frac{3}{2}c^2 = 1$$

$$c = \sqrt{\frac{2}{3}}$$

1.11 Slide 32

$$E[X] = \int_c^{2c} x \cdot x dx = \frac{x^3}{3} \Big|_c^{2c} = \frac{7}{3}c^3 = 1.27$$

$$E[X^2] = \int_c^{2c} x^2 \cdot x dx = \frac{x^4}{4} \Big|_c^{2c} = \frac{15}{4}c^4 = \frac{5}{3}$$

$$\text{var}(X) = \frac{5}{3} - 1.27^2 = 0.0538$$

1.12 Slide 34

$$F_X(x) = \begin{cases} 0 & x \leq c \\ \frac{x^2 - c^2}{2} & c < x \leq 2c \\ 1 & x > 2c \end{cases}$$

1.13 Slide 36

$$y = \exp(x)$$

$$x = \ln(y)$$

$$F_Y(y) = \int_0^{\ln y} p_X(x) dx$$

$$= \ln y$$

$$f_Y(y) = \frac{dF_Y(y)}{dy}$$

$$= \frac{1}{y} \text{ for } 1 < y < e$$