Name: ______________________________ ID #: ______________________________

CENG 114 – Probability and Statistical Methods for Engineers

Professor Shyue Ping Ong

Mid-term Exam 1

Winter, Jan 30 2018

Instructions:

1. Write your name on the top right of this page.
2. Please answer **ALL** questions as far as possible.
3. You have a total of 80 mins exactly.
4. Write your answers in the space provided following the questions. If you need additional space, you may request for additional blank sheets of paper during the exam. If you use additional blank sheets, you should mark the top left of each sheet with your name, ID#, question number, and today's date, e.g., "John Smith, A1234567, Q1, Jan 30 2018".
5. Indicative maximum points for each sub-question are provided only as a reference. Actual grading points may vary.
6. Write neatly and legibly. Illegible answers will be considered as wrong.

Question 1 (20 points): ____________

Question 2 (20 points): ____________

Question 3 (20 points): ____________

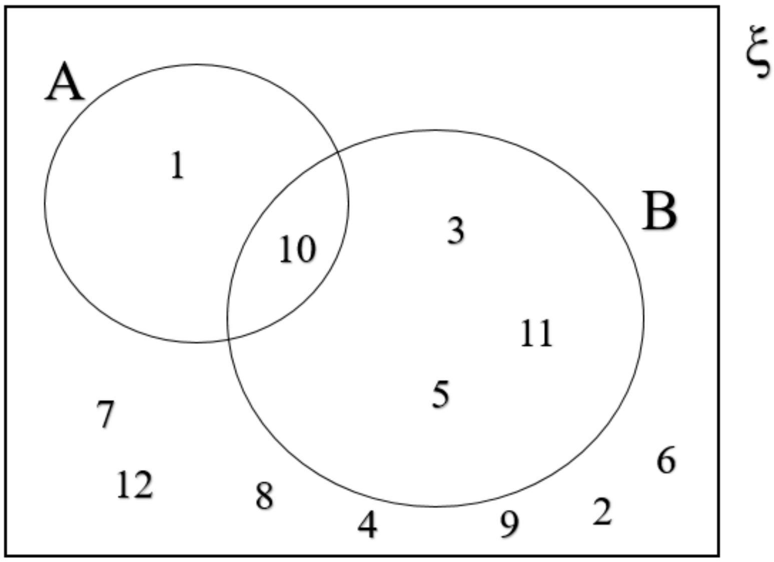
Question 4 (20 points): ____________

Question 5 (20 points): ____________

This exam has a total of 8 pages, including the cover page.

Qn 1.

The Venn diagram below illustrates sets A and B.



For each of the following scenarios, write out the result in set notation and calculate the corresponding probability associated with each of these:

i. (4 points) $A \cap B$

ii. (4 points) B^C

iii. (4 points) $A \cup B$

iv. (4 points) $A^C \cap B^C$

v. (4 points) $A^C \cup B$

Solutions

i. $A \cap B = \{10\}$

$P(A \cap B) = 1/12 = .083$

ii. $B^C = \{1,2,4,6,7,8,9,12\}$

$P(B^C) = 8/12 = 2/3 = 0.6666$

iii. $A \cup B = \{1,3,5,10,11\}$

$P(A \cup B) = 5/12 = 0.416$

iv. $A^C \cap B^C = \{2,4,6,7,8,9,12\}$

P ($A' \cap B'$) = 7/12 = 0.583

v. $A^C \cup B$ = {2,3,4,5,6,7,8,9,10,11,12}

P ($A^C \cup B$) = 11/12 = 0.917

Qn 2.

The alignment between the magnetic media and head in a magnetic storage system affects the system's performance. Suppose that 10% of the read operations are degraded by skewed alignments, 5% of the read operations are degraded by off-center alignments, and the remaining read operations are properly aligned. The probability of a read error is 0.01 from a skewed alignment, 0.02 from an off-center alignment, and 0.001 from a proper alignment.

i. (3 points) What is the probability of a read error?

ii. (8 points) If a read error occurs, what is the probability that it is due to a skewed alignment?

iii. (6 points) Three read operations were performed in sequence. It is known that the first operation was performed on a skewed alignment, the second operation was performed an off-center alignment, and the final operation was performed on a proper alignment. What is the probability that there are no errors in the three read operations?

iv. (3 points) Five operations were performed. What is the probability that there is at least one degraded operation?

Solutions:

i. Let us define the following events:

E: Error

S: Skewed

O: Off-center

P: proper alignment

$$P(E) = P(E|S)P(S) + P(E|O)P(O) + P(E|P)P(P)$$
$$= 0.01 \times 0.1 \; + \; 0.02 \times 0.05 \; + \; 0.001 \times 0.85$$
$$= 0.00285$$

ii. $P(S|E) = \frac{P(E|S)P(S)}{P(E)} = \frac{0.01 \times 0.1}{0.00285} = 0.351$

iii. $P(No\ errors) = 0.99 \times 0.98 \times 0.999 = 0.969$

iv. $P(at\ least\ one\ degraded) = 1 - P(0\ degraded) = 1 - 0.85^5 = 0.556$

Qn 3.

There are 3 red balls, 2 green balls, and 5 blue balls. There are no distinguishing marks on any of the balls other than the color.

i. (6 points) How many ways are there of arranging the balls in a sequence?

ii. (4 points) How many ways are there of arranging the balls so that balls of the same color are together?

iii. (6 points) All 10 balls are put in a bag. One ball is taken out at a time at random without replacement. What is the probability that the first three balls taken out are red?

iv. (4 points) All 10 balls are put in a bag. One ball is taken out at a time at random without replacement. How many balls must be taken out for there to be a greater than 60% probability of having at least one red ball?

Solutions:

i. $\frac{10!}{3!*2!*5!} = 2520$

ii. $3! = 6$

iii. Applying the multiplication rule,

$$p = P(1st\ red)P(2nd\ red\ |1st\ red)P(3rd\ red\ |1st\ and\ 2nd\ red)$$

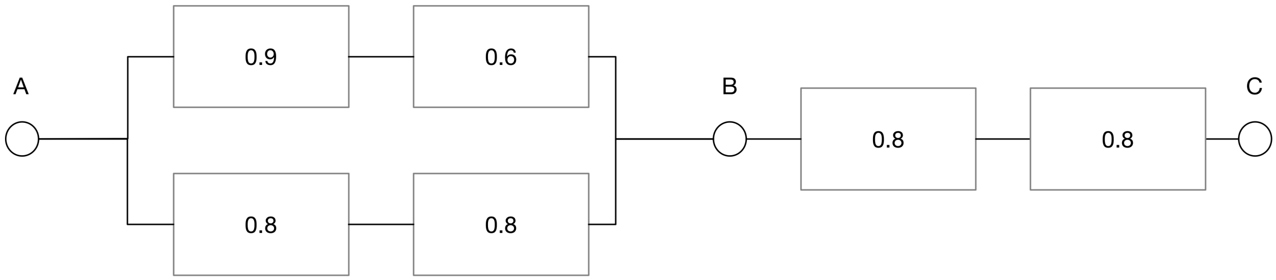
$$= \frac{3}{10}\frac{2}{9}\frac{1}{8} = 0.0083$$

iv. The probability of getting no red ball is

$$p = 1 - \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \times \ldots$$

This sequence becomes > 0.6 when you have taken 3 balls.

Qn 4.



A signal network is shown above, and the number in the boxes indicate whether a particular repeater is working. Assuming that each repeater fails independently unless otherwise stated, answer the following questions:

i. (8 points) What is the probability that there is a working path between A and B?

ii. (4 points) What is the probability that there is a working path between A and C?

iii. (4 points) What is the probability that there is a working path between A and C, given that there is a working path between A and B?

iv. (4 points) It turns out that the two repeaters between B and C are located in the same geographical location, i.e., their failure rates are not completely independent due to common factors such as electrical outages, acts of god, etc. The probability that both repeaters fail is 0.1. What is the probability that there is a working path between B and C in this case?

Solutions:

i. $$1-(1-0.9\times 0.6)(1-0.8\times 0.8)=0.834$$

ii. $$0.834\times 0.8\times 0.8=0.534$$

iii. This is simply equals to the probability that there is a working path between B and C, since the repeaters are independent. So 0.8 x 0.8 = 0.64

iv. Let F1 and F2 denote the events that the 1st and 2nd repeaters between B and C fail, respectively.

$$P(F1)=1-0.8=0.2$$
$$P(F2)=1-0.8=0.2$$
$$P(F1\cap F2)=0.1$$
$$P(F1\cup F2)=P(F1)+P(F2)-P(F1\cap F2)=0.3$$
$$P(B\to C)=1-P(F1\cup F2)=0.7$$

Qn 5.

A biased 6-sided dice manufactured by a company has the interesting property that the probability of showing a number is proportional to the number itself, i.e., the dice has twice the probability of showing a 2 versus showing a 1, and three times the probability of showing a 3 versus showing a 1, etc.

i. (4 points) What is the probability that the biased dice shows a one?

ii. (2 points) What is the probability that the biased dice shows a six?

iii. (4 points) The biased dice is rolled 5 times. What is the probability that there is at least one six among the 5 rolls?

iv. (4 points) Two biased dice and three unbiased dice (which have equal probability of showing any of the numbers) is put in a bag. A person selects a dice from the bag at random and rolls. What is the probability that the roll is not a 6?

v. (6 points) The same experiment is carried out as part (iv). Given that the result of a roll is not a 6, what is the probability that the selected dice is biased?

Solutions:

i. Let the probability that the dice show a 1 be k. Using the normalization axiom,

$$k + 2k + 3k + 4k + 5k + 6k = 1$$

$$k = \frac{1}{21} = 0.048$$

ii.

$$P(6) = 6k = \frac{2}{7} = 0.286$$

iii.

$$P(6^C) = 1 - \frac{2}{7} = \frac{5}{7}$$

$$P\left(at\ least\ one\ six\ in\ 5\ rolls\right) = 1 - P\left(no\ 6\ in\ 5\ rolls\right)$$

$$= 1 - \left(\frac{5}{7}\right)^5 = 0.814$$

iv. biased: $\frac{2}{5} * \frac{15}{21} = 0.286$

unbiased: $\frac{3}{5} * \frac{5}{6} = 0.5$

The probability that the roll is not a 6 = 0.786

v. Bayes rule:

P(biased | not 6) = P(not 6 | biased) * P(biased) / P(not 6) = 0.286/0.786 = 0.364