Challenge Set 4

Deadline: Mar 8 2018 at 5pm

Challenge problems are **optional** problems for those interested in testing their abilities. For each correct answer to a challenge question, bonus points of 0.3 are given towards the final overall grade, i.e., you can potentially earn up to 4.5 points towards the final grade if you get all questions correct (note that 4.5 points on your final grade is a non-trivial amount of points because no scaling ratio is applied). Proper workings must be shown to get any points, and there is no partial credit. Also, because these are bonus questions, instructors will not provide any help or hints (unlike typical problem or practice set questions where generous assistance will be provided) to be fair to all students. Please submit your solutions to SME office 244E or during lectures in person by the deadline.

Q1. The Stefan-Boltzmann law states that the total power P radiated from a black/grey body is given by

$$P = A\varepsilon\sigma T^{b}$$

where A is the surface area of the body, ε is the dimensionless emissivity that is always ≤ 1, σ is the Stefan-Boltzmann constant that has a value of 5.670×10^{-8} W m^{-2} K^{-4}, and b is a constant. An experiment is carried out by measuring the power irradiated from a sphere made up of a single material of diameter 10 cm. The data collected is presented in the table below. Blank columns and rows are provided for your workings if you wish to use them.

P (W)	T (K)
12.6	300
23.5	350
42.2	400
66.9	450
100.4	500
146.6	550
208.9	600
286.7	650
386.1	700

i. (0.2 points) By performing a linear least squares regression, determine the emissivity of the material to 3 significant figures and the constant b to the nearest integer.

ii. (0.2 points) The temperature can only be controlled to an uncertainty of ±20 K. What is the uncertainty in the power calculated at a temperature of 800K? You may ignore the uncertainties in your estimate of ε and b.

iii. (0.2 points) Sometimes, the power is measured to estimate the temperature of a body. The sun can be considered as an ideal black body with $\varepsilon = 1$. The radius of the sun is $(6.957 \pm 0.06) \times 10^{8}$ m and the power irradiated by the sun is estimated to be $(3.828 \pm 0.05) \times 10^{26}$ W. Estimate the temperature of the sun and provide an uncertainty for your estimate as well. You should use the value of b determined previously, and ignore any uncertainty in b.

Solutions:

i.

P (W)	T (K)	Y = ln(P)	X = ln(T)	XY
12.6	300	2.534	5.704	14.452
23.5	350	3.157	5.858	18.493
42.2	400	3.742	5.991	22.423
66.9	450	4.203	6.109	25.678
100.4	500	4.609	6.215	28.644
146.6	550	4.988	6.310	31.472
208.9	600	5.342	6.397	34.171
286.7	650	5.658	6.477	36.650
386.1	700	5.956	6.551	39.019

$\ln \mathrm{P} = \ln(\mathrm{A}\varepsilon\sigma) + b \ln \mathrm{T}$ or $\mathrm{Y} = a + b\mathrm{X}$

$$\mathrm{SS_X} = \sum \mathrm{X}^2 - \frac{(\sum \mathrm{X})^2}{9} = 0.662$$

$$\mathrm{SP} = \sum \mathrm{XY} - \frac{\sum \mathrm{X} \sum \mathrm{Y}}{9} = 2.666$$

$$b = \frac{\mathrm{SP}}{\mathrm{SS_X}} = 4.03 \sim 4$$

$$a = \overline{\mathrm{Y}} - b\overline{\mathrm{X}} = 4.465 - 4.03 * 6.179 = -20.42$$

$$\mathrm{A} = 4\pi \mathrm{r}^2 = 4\pi(0.05)^2 = 0.0314$$

$$\varepsilon = \frac{e^a}{\mathrm{A}\sigma} = 0.760\ (K^4)$$

ii.

$$\frac{\partial P}{\partial T} = 4A\varepsilon\sigma T^3 = 4 * 4\pi(0.05)^2 * 0.76 * 5.67 * 10^{-8} * 800^3 = 2.773$$

$$\sigma_\mathrm{T} = 20$$

$$\sigma_\mathrm{P} = \sqrt{\left(\frac{\partial \mathrm{P}}{\partial \mathrm{T}}\right)^2 {\sigma_\mathrm{T}}^2} = 55.46$$

iii.

$r \sim 6.957 * 10^8$; $P \sim 3.828 * 10^{26}$; $\sigma_r = 0.06 * 10^8$; $\sigma_P = 0.05 * 10^{26}$;
$\sigma = 5.670 * 10^{-8}$
$P = A\varepsilon\sigma T^4$

$$\rightarrow T = \frac{P^{0.25}}{A^{0.25} * \sigma^{0.25}} = \frac{P^{0.25}}{(4\pi r^2)^{0.25} * \sigma^{0.25}} = 5772.1\ (K)$$

$$\sigma_T = \sqrt{\left(\frac{\partial T}{\partial r}\right)^2 \sigma_r{}^2 + \left(\frac{\partial T}{\partial P}\right)^2 \sigma_P{}^2}$$

$$\frac{\partial T}{\partial P} = \frac{0.25 * P^{-0.75}}{A^{0.25} * \sigma^{0.25}} = 3.77 * 10^{-24}$$

$$\frac{\partial T}{\partial r} = \frac{\partial}{\partial r}\left(\frac{P^{0.25} * r^{-0.5}}{(4\pi)^{0.25} * \sigma^{0.25}}\right) = \left(\frac{-0.5 * P^{0.25} * r^{-1.5}}{(4\pi)^{0.25} * \sigma^{0.25}}\right) = -4.148 * 10^{-6}$$

$\rightarrow \sigma_T = 31.22$

Q2. You are testing pH dependency of a new synthesis technique to make gold nanoparticles. You try solvents of different pH to use as your dispersing solution and measure your concentrated NP yield after synthesis. The results from your experiments are presented in the following table.

pH	4.6	4.8	5.2	5.4	5.6	5.8	6.0
Yield (mL)	1056	1833	1629	1852	1783	2647	2131

i. (0.1 pts) If the pH is increased by 0.1, by how much would you predict the yield to increase or decrease?

ii. (0.1 pts) Predict the yield for a pH of 5.5.

iii. (0.1 pts) For what pH would you predict a yield of 1500 pounds per acre?

Solutions:

The equation of the least-squares line is $y = -2090.942029 + 737.101449x$.

i. 737.101449(0.1) = 73.71. Increase by 73.71 pounds per acre.

ii. −2090.942029+737.10144(5.5) = 1963.1 hours

iii. Let x be the required pH.

Then $1500 = -2090.942029 + 737.10144x$

so $x = 4.872$.