Challenge Set 3

Deadline: Feb 20 2018 at 5pm

Challenge problems are **optional** problems for those interested in testing their abilities. For each correct answer to a challenge question, bonus points of 0.3 are given towards the final overall grade, i.e., you can potentially earn up to 4.5 points towards the final grade if you get all questions correct (note that 4.5 points on your final grade is a non-trivial amount of points because no scaling ratio is applied). Proper workings must be shown to get any points, and there is no partial credit. Also, because these are bonus questions, instructors will not provide any help or hints (unlike typical problem or practice set questions where generous assistance will be provided) to be fair to all students. Please submit your solutions to SME office 244E or during lectures in person by the deadline.

Q1. You attend a chemical engineering conference with 500 total attendees, including presenters. You are curious during breaktime and decide to calculate the probability that exactly one other conference attendee has the same birthday as you. Calculate this probability (a) (0.2 points) explicitly and (b) (0.1 points) approximately via the Poisson distribution. You may ignore leap years.

Q2. A source transmits binary 1 as a +1V with probability p and binary 0 as -1V with probability $1 - p$. Noise in the transmission channel follows the following distribution:

$$f(n) = kn^2 \text{ for -m} < n < \text{m where } k \text{ and } m \text{ are constants.}$$

At the destination, the signal is interpreted as binary 1 if it falls above a certain threshold voltage T and binary 0 otherwise. Derive an expression for T in terms of p that minimizes the total probability of error. (0.3 points)

Q3. Assume the distribution of trip times for a staff from his home to office follows normal distribution and the average time and standard deviation are 24 minutes and 3.8 minutes, respectively. Find the following:

(a) (0.1 points) What is the probability that a trip will take 30 minutes at least?

(b) (0.1 points) If the office opens at 8:00 A.M. and the staff leaves his house at 7:45 A.M. daily, what percentage of the time is he late for work?

(c) (0.1 points) If he leaves the house at 7:35 A.M. and coffee is served at the office from 7:50 A.M. until 8:00 A.M., what is the probability that he misses coffee?