

Challenge Set 2

Deadline: Feb 9 2018 at 5pm

Challenge problems are **optional** problems for those interested in testing their abilities. For each correct answer to a challenge question, bonus points of 0.3 are given towards the final overall grade, i.e., you can potentially earn up to 4.5 points towards the final grade if you get all questions correct (note that 4.5 points on your final grade is a non-trivial amount of points because no scaling ratio is applied). Proper workings must be shown to get any points, and there is no partial credit. Also, because these are bonus questions, instructors will not provide any help or hints (unlike typical problem or practice set questions where generous assistance will be provided) to be fair to all students. Please submit your solutions via email (you can simply take a good resolution photo/scan of your solutions with your student ID number and name clearly labelled) to yul277@eng.ucsd.edu and t2washin@eng.ucsd.edu by the deadline.

Q1. As a new lab technician, you have synthesized nanorods with length in the range of 101nm to 110nm in 1nm increments, i.e., possible values of random variable are 101, 102, ..., 110, with equal probability. To decrease the length of nanorods for use in a separate experiment, you synthesized nanorods on three different days to optimize your procedure, and select the batch that has the minimum length X among the three days, i.e., $X = \min(X_1, X_2, X_3)$, where X_1 , X_2 , and X_3 are the lengths of the nanorods of days 1, 2 and 3, respectively. Assume that the synthesized lengths of each day are independent of that on other days.

- i. What is the PMF of X ? (0.2 pts)
- ii. What is the expected decrease in the nanorod length as a result of this minimum-of-three-days synthesis procedure, versus a one-off synthesis? (0.1 pts)

Q2. Two coins are simultaneously tossed until one of them comes up a head and the other a tail. The first coin comes up a head with probability p and the second with probability q . All tosses are assumed independent.

- i. Find the PMF, the expected value, and the variance of the number of tosses. (0.2 pts)
- ii. What is the probability that the last toss of the first coin is a head? (0.1 pts)

Q3. Consider the following PDF:

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

- i. Calculate k . (0.1 pts)
- ii. Derive $F(x)$. (0.1 pts)
- iii. Calculate $P(0.3 < x < 1.6)$. (0.1 pts)