Name: ______________________________ ID #: ______________________________

CENG 114 – Probability and Statistical Methods for Engineers

Professor Shyue Ping Ong

Mid-term Exam 2

Winter 2017, Feb 23 2017

Instructions:

1. Write your name on the top right of this page.
2. Please answer **ALL** questions as far as possible.
3. You have a total of 80 mins exactly.
4. Write your answers in the space provided following the questions. If you need additional space, you may request for additional blank sheets of paper during the exam. If you use additional blank sheets, you should mark the top left of each sheet with your name, ID#, question number, and today's date, e.g., "John Smith, A1234567, Q1, Feb 2 2017".
5. Indicative maximum points for each sub-question are provided only as a reference. Actual grading points may vary.
6. Write neatly and legibly. Illegible answers will be considered as wrong.
7. You may detach the supplemental appendices (standard normal tables, etc.) at the end of this exam and do not need to hand these in.

Question 1 (25 points): ____________

Question 2 (25 points): ____________

Question 3 (25 points): ____________

Question 4 (25 points): ____________

This exam has a total of 10 pages, including the cover page.

Qn 1.

The joint probability density function of two random variables *X* and *Y* are given below:

$$f_{X,Y}(x,y) = \begin{cases} kx^2(1+y) & \text{for } 0 < x < 1, -1 < y < 0 \\ 0 & \text{otherwise} \end{cases}$$

where *k* is a constant. Answer the following questions.

i. (5 points) Determine the value of *k* that will make this a valid PDF.

ii. (5 points) Determine the marginal PDFs of *X* and *Y*.

iii. (1 point) Are *X* and *Y* independent?

iv. (8 points) Calculate the expected values and variances of *X* and *Y*.

v. (1 point) Calculate cov(*X*, *Y*).

vi. (5 points) A random variable *Z* is given by *Z* = exp(*X*). Determine the PDF of *Z*.

Qn 2.

Potassium-argon (K-Ar) dating is a radiometric dating method using in geochronology and archaeology. K is common in many materials. The radioactive isotope ^{40}K decays to ^{40}Ca in 89.1% of decay events, while conversion to stable ^{40}Ar (via electron capture) occurs in the remaining 10.9% of events. ^{40}Ar is a noble gas and remains trapped after decay from ^{40}Ca. The half-life of ^{40}K is 1.248×10^9 years.

Answer the following questions:

i. (2 points) What is the rate constant of the radioactive decay process in year^{-1}?

ii. (6 points) The Mars Curiosity Rover used the K-Ar dating method to date a piece of rock nicknamed "Cumberland" on the Martian surface. Analysis of the rock showed that for every 10,000 ^{40}K atoms, there are 2.339 ^{40}Ar atoms. Assuming that the rock is a closed system, i.e., no atoms have entered or left the rock since the beginning of its life and there was no ^{40}Ar in the rock when it was first formed, estimate the ratio of the amount of ^{40}K at the beginning of its life to the amount of ^{40}K today. Giving your answers to 5 decimal places.

iii. (7 points) Estimate the age of the rock in part (ii).

iv. (10 points) Another rock sample is estimated to be 4.56 million years old. Estimate the ratio of ^{40}Ar to ^{40}K in the rock today.

Qn 3.

The number of requests for a particular web page can be modelled as a Poisson process with an average of 2 requests per minute. Answer the following questions, giving your answer to three significant figures.

i. (6 points) What is the probability that there are more than 2 requests in any given minute?

ii. (4 points) What is the probability that there are no requests for the web page in two minutes?

iii. (6 points) Estimate the probability that there are less than 2800 requests for the web page in a day.

iv. (9 points) The webmaster decides that he will redesign the page if there are less than 2800 requests per day for more than 30 days in a year (assume it is a standard year with 365 days). Estimate the probability that he will redesign the page.

Qn 4.

A semiconductor manufacturer makes silicon wafers with thicknesses that are normally distributed with a mean of 2 inches. 80% of the produced wafers are between 1.97 and 2.03 inches thick.

i. (4 points) What is the standard deviation of the thickness of the wafers? Give your answer to 3 significant figures.

ii. (4 points) Wafers are scrapped if they are greater than 2.05 inches or less than 1.93 inches. What is the probability that a wafer will get scrapped? Give your answers to 3 significant figures.

iii. (6 points) What is the probability that exactly 4 wafers out of 6 will be between 1.99 and 2.02 inches?

iv. (6 points) For a particular project, wafers with thickness greater than 2.01 inches are needed. An engineer picks wafers one at a time to test until a wafer meeting the requirement is found. What is the probability that the 7^{th} wafer picked is the first one to meet the requirement?

v. (5 points) For the same project as (iv), the engineer needs 3 wafers meeting the requirement. What is the average number of wafers the engineer needs to test before finding the necessary number of wafers?

Appendix 1: Standard Normal Cumulative Probability Table

Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998