

Challenge Set 3

Deadline: Feb 21 2017 at 5pm

Challenge problems are *optional* problems for those interested in testing their abilities. For each correct answer to a challenge question, bonus points of 0.1 are given towards the *final overall grade*, i.e., you can potentially earn up to 1.5 points towards the final grade if you get all questions correct. Proper workings must be shown to get any points, and there is no partial credit. Also, because these are bonus questions, instructors will not provide any help or hints (unlike typical problem or practice set questions where generous assistance will be provided) to be fair to all students. Please submit your solutions via the Turnitin assignment “Challenge Set 3” on TED@UCSD (you can simply take a good resolution photo/scan of your solutions with your student ID number and name clearly labelled and convert it to a PDF for upload) by the deadline.

Q1: Pat and Nat are dating, and all of their dates are scheduled to start at 9 pm. Nat always arrives promptly at 9 pm. Pat is highly disorganized and arrives at a time that is uniformly distributed between 8 pm and 10 pm. Let X be the time in hours between 8 pm and the time when Pat arrives. If Pat arrives before 9 pm, their date will last exactly 3 hours. If Pat arrives after 9 pm, their date will last for a time that is uniformly distributed between 0 and $3 - X$ hours. The date starts at the time they meet. Nat gets irritated when Pat is late and will end the relationship after the second date on which Pat is late by more than 45 minutes. All dates are independent of any other dates.

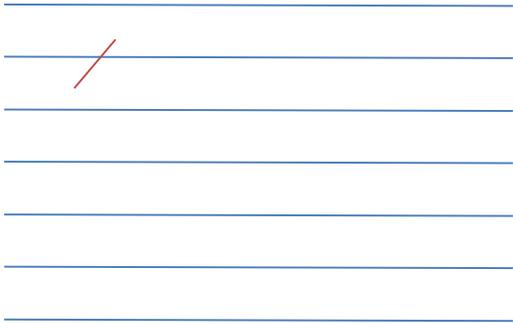
- (a) What is the expected number of hours Nat waits for Pat to arrive?
- (b) What is the expected duration of any particular date?
- (c) What is the expected number of dates they will have before breaking up?

Q2: Two machines are producing parts for car engines. The first machine can produce 120 parts per hour, but it makes an average of 1 mistake per hour. The second machine can produce 150 parts per hour, but it makes an average of 3 mistakes per hour. The two machines function independently.

- a. What is the probability that during a two-hour period, six total mistakes are made?
- b. The factory shuts down when 6 mistakes are made in a two-hour shift. About how many consecutive shifts can there be with a 5% chance of shutting down on the final shift?

Q3: A wooden rod of length $2a$ units is dropped at random onto a surface covered with infinitely long parallel metal rods (distance between the infinitely long parallel metal rods is also $2a$ units). Find the probability that wooden rod touches one of the metal rods.

Figure



Blue lines indicate infinitely long metal rods (spacing between the blue lines is $2a$ units) and orange line indicates the wooden rod.