

## Challenge Set 2

*Deadline: Feb 14 2017 at 5pm*

Challenge problems are **optional** problems for those interested in testing their abilities. For each correct answer to a challenge question, bonus points of 0.1 are given towards the *final overall grade*, i.e., you can potentially earn up to 1.5 points towards the final grade if you get all questions correct. Proper workings must be shown to get any points, and there is no partial credit. Also, because these are bonus questions, instructors will not provide any help or hints (unlike typical problem or practice set questions where generous assistance will be provided) to be fair to all students. Please submit your solutions by going to the TritonEd System -> Content -> Challenge Set 2 (which is a TurnItIn assignment) by the deadline. You can simply take a good resolution photo/scan of your solutions with your student ID number and name clearly labelled and convert it to a PDF for upload.

**Q1:** The following is a valid probability distribution function of a continuous random variable from  $0 < x < 1$ . A and B are constants.

$$f_X(x) = A + Bx^3$$

Given that  $E[X] = \frac{2}{3}$ , find the numerical value of the variance. Your answer should NOT be in terms of A and B.

**Q2:** A fair coin is tossed repeatedly until two heads or two tails appear consecutively. Derive the probability mass function, expected value and variance of the number of tosses.

**Q3:** A PDF of a random variable is given by:

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|} \text{ for } -\infty < x < \infty$$

- i. Demonstrate that  $f_X(x)$  satisfies the normalization condition.
- ii. Calculate the mean and variance of  $X$ .