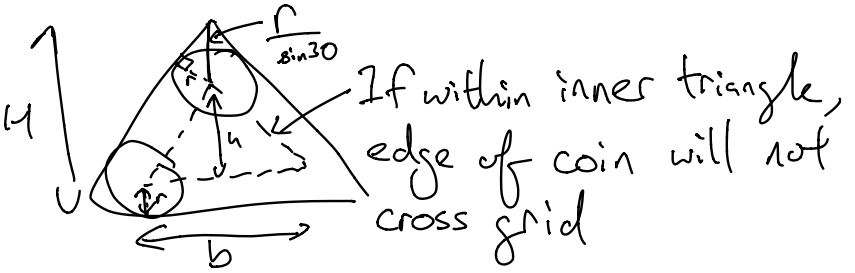
Challenge Set 1 Solutions

Q1:



$$H = \sqrt{10^2 - 5^2} = 5\sqrt{3}$$

$$h = H - \frac{1}{\sin 30^\circ} - 1 = 5\sqrt{3} - 3$$

$$A = \frac{1}{2} 5\sqrt{3} \times 10 = 25\sqrt{3}$$

$$\frac{\frac{1}{2}b}{h} = \tan 30^\circ \Rightarrow b = (5\sqrt{3} - 3)\frac{1}{\sqrt{3}} \times 2$$

$$= (5 - \sqrt{3})2$$

$$\text{area of inner triangle} = (5 - \sqrt{3})(5\sqrt{3} - 3)$$

$$= 25\sqrt{3} - 15 - 15 + 3\sqrt{3}$$

$$= 28\sqrt{3} - 30$$

$$P(\text{no crossing}) = \frac{\text{Area of } \triangle}{\text{Area of } \triangle} = \frac{28\sqrt{3} - 30}{25\sqrt{3}} = 0.427$$

Q2:

Case 1: No replacement

Take the number of suits times the number of ways to get 5 cards of one suit divided by the total number of 5 card hands.

$(4 * {}_{13}C_5)/({}_{52}C_5) = 0.001981$

Case 2: Replace 1 card

Take the number of suits times the number of ways to get 4 cards of one suit times the number of ways to get 1 card of a different suit divided by the total number of 5

card hands. Multiply this by the number of ways to get one of the remaining 9 cards of that suit divided by the number of ways to get one of the remaining total 47 cards.

$\{(4 * {}_{13}C_4 * {}_{39}C_1)/({}_{52}C_5)\}*\{{}_9C_1/{}_{47}C_1\} = 0.008218$

Case 3: Replace 2 cards

Similar procedure as Case 2 with different numbers.

$\{(4 * {}_{13}C_3 * {}_{39}C_2)/({}_{52}C_5)\}*\{{}_{10}C_2/{}_{47}C_2\} = 0.013578$

Case 3: Replace 3 cards

Similar again.

$\{(4 * {}_{13}C_2 * {}_{39}C_3)/({}_{52}C_5)\}*\{{}_{11}C_3/{}_{47}C_3\} = 0.011164$

Add them all up: 0.001981 + 0.008218 + 0.013578 + 0.011164 = 0.034941

Q3:
Answer=1- (Probability that D5 does not show a number that appears on D1 D2 or D3 or D4).
Let's calculate Probability of D5 not showing a number that appears on D1 or D2 or D3 or D4

There are six possibilities for D5. Once a number is fixed on D5. D1 D2 D3 and D4 can have only 5 possibilities each.
So total number of favorable outcomes= 6*5*5*5*5
Total number of outcomes for 3 Dice is 6*6*6*6*6
So Answer is **1-(6*5*5*5*5)/ (6*6*6*6*6) = 0.5177**