**Q1**

(a) $\overline{X} = 180$, $s = 14$, $n = 120$, $z_{.025} = 1.96$.

The confidence interval is $180 \pm 1.96\left(14/\sqrt{120}\right)$, or (177.495, 182.505).

(b) $\overline{X} = 180$, $s = 14$, $n = 120$, $z_{.005} = 2.58$.

The confidence interval is $180 \pm 2.58\left(14/\sqrt{120}\right)$, or (176.703, 183.297).

(c) $\overline{X} = 180$, $s = 14$, $n = 120$, so the upper confidence bound 182 satisfies

$182 = 180 + z_{\alpha/2}\left(14/\sqrt{120}\right)$.

Solving for $z_{\alpha/2}$ yields $z_{\alpha/2} = 1.56$.

The area to the right of $z = 1.56$ is $1 - 0.9406 = 0.0594$, so $\alpha/2 = 0.0594$.

$$\text{The level is } 1 - \alpha = 1 - 2(0.0594) = 0.8812, \text{ or } 88.12\%.$$

(d) $z_{.025} = 1.96$. $1.96\left(14/\sqrt{n}\right) = 2$, so $n = 189$.

(e) $z_{.005} = 2.58$. $2.58\left(14/\sqrt{n}\right) = 2$, so $n = 327$.

Q2

(a) $\overline{X} = 1.57$, $s = 0.1$, $n = 80$, $z_{.025} = 1.96$.

The confidence interval is $1.57 \pm 1.96\left(0.1/\sqrt{80}\right)$, or (1.5481, 1.5919).

(b) $\overline{X} = 1.57$, $s = 0.1$, $n = 80$, $z_{.01} = 2.33$.

The confidence interval is $1.57 \pm 2.33\left(0.1/\sqrt{80}\right)$, or (1.5439, 1.5961).

(c) $\overline{X} = 1.57$, $s = 0.1$, $n = 80$, so the upper confidence bound 1.59 satisfies

$1.59 = 1.57 + z_{\alpha/2}\left(0.1/\sqrt{80}\right)$.

Solving for $z_{\alpha/2}$ yields $z_{\alpha/2} = 1.79$.

The area to the right of $z = 1.79$ is $1 - 0.9633 = 0.0367$, so $\alpha/2 = 0.0367$.

$$\begin{aligned}\text{The level is } 1 - \alpha &= 1 - 2(0.0367) \\ &= 0.9266, \text{ or } 92.66\%.\end{aligned}$$

(d) $z_{.025} = 1.96$. $1.96\left(0.1/\sqrt{n}\right) = 0.01$, so $n = 385$.

(e) $z_{.01} = 2.33$. $2.33\left(0.1/\sqrt{n}\right) = 0.01$, so $n = 543$.

Q3

$\overline{X} = 5.700$, $s = 0.40000$, $n = 6$, $t_{6-1,.025} = 2.571$.

The confidence interval is $5.7 \pm 2.571\left(0.40000/\sqrt{6}\right)$, or (5.280, 6.120).

Q4

$\overline{X} = 482.6857$, $s_X = 14.046892$, $n_X = 14.0$, $\overline{Y} = 464.6556$, $s_Y = 14.33389$, $n_Y = 9.0$.

The number of degrees of freedom is

$$\nu = \frac{\left[\dfrac{14.046892^2}{14.0} + \dfrac{14.33389^2}{9.0}\right]^2}{\left[\dfrac{(14.046892^2/14.0)^2}{14.0-1} + \dfrac{(14.33389^2/9.0)^2}{9.0-1}\right]}$$

= 16, rounded down to the nearest integer.

$t_{16,\,0.01} = 2.583$, so the confidence interval is

$$482.6857 - 464.6556 \pm 2.583\sqrt{\frac{14.046892^2}{14.0} + \frac{14.33389^2}{9.0}},\ \text{or } (2.335, 33.725).$$

Q5:

Given:

Mean daily production, $\mu = 740$ tons
Number of days, $n = 60$
Sample mean, $\overline{X} = 735$ tons/day
Standard deviation, $s = 25$ tons/day

The null and alternate hypotheses are $H_0: \mu \geq 740$ versus $H_1: \mu < 740$.

The z−score, $z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}}$

$$= \frac{(735 - 740)}{25/\sqrt{60}}$$

$= -1.55$

Since the alternate hypothesis is of the form $\mu < \mu_0$, the $P$−value is the area to the left of $z = -1.55$.

Thus, from the table given below P = 0.0606.

Q6:

(a) $\overline{X} = 3.23$, $s = 0.51$, $n = 6$.
There are $6 - 1 = 5$ degrees of freedom.
$t = (3.23 - 3)/(0.51/\sqrt{6}) = 1.1047$.
Since the alternate hypothesis is of the form $\mu > \mu_0$, the $P$–value is the area to the right of $t = 1.1047$. From the $t$ table, $0.10 < P < 0.25$. A computer package gives $P = 0.1598$.
We cannot conclude that the mean surface deflection is greater than 3 mm.
(b) $\overline{X} = 3.23$, $s = 0.51$, $n = 6$.
There are $6 - 1 = 5$ degrees of freedom.
$t = (3.23 - 2.5)/(0.51/\sqrt{6}) = 3.5061$.
Since the alternate hypothesis is of the form $\mu > \mu_0$, the $P$–value is the area to the right of $t = 3.5061$. From the $t$ table, $0.005 < P < 0.01$. A computer package gives $P = 0.00859$.
We can conclude that the mean surface deflection is greater than 2.5 mm.

Q7:

For dye $A$:

$n_x = 4$

$$\overline{X} = \sum_{i=1}^{4} \frac{X_i}{4} = \frac{(1.1 + 1.7 + 1.1 + 3.3)}{4} = 1.8$$

$$s_x = \sqrt{\frac{\sum_{i=1}^{4} (x_i - \overline{x})^2}{(4 - 1)}}$$

$$= \sqrt{\frac{(1.1 - 1.8)^2 + (1.7 - 1.8)^2 + (1.1 - 1.8)^2 + (3.3 - 1.8)^2}{4 - 1}}$$

$= 1.03923$

Similarly for dye $B$, $n_y = 4, \overline{Y} = 3.05, s_y = 0.60277$

The numbers of degrees of freedom is

$$\nu = \frac{\left[\frac{1.03923^2}{4} + \frac{0.60277^2}{4}\right]^2}{\frac{(1.03923^2/4)^2}{4-1} + \frac{(0.60277^2/4)^2}{4-1}} = 4\text{, rounded down to nearest integer.}$$

$$t_3 = \frac{(1.8 - 3.05 - 0)}{\sqrt{\frac{1.03923^2}{4} + \frac{0.60277^2}{4}}} = -2.081$$

The null and hypotheses are $H_0: \mu_X - \mu_Y \geq 0$ versus $H_1: \mu_X - \mu_Y < 0$.

Since, the alternate hypothesis is of the form $\mu_X - \mu_Y < \Delta$, the $P$−value is the area to the left of $t = -2.081$.

From the table given below, the $P$−value is greater than the threshold value of 0.05. Thus, we cannot conclude that the mean strength of crayons made with dye$B$ is greater than that made with dye $A$ .

Q8:

For dye $A$:

$n_X = 4$

$$\overline{X} = \sum_{i=1}^{4} \frac{X_i}{4} = \frac{(1.2 + 3.4 + 1.9 + 3.2)}{4} = 2.4$$

$$s_X = \sqrt{\frac{\sum_{i=1}^{4} (x_i - \overline{x})^2}{(4-1)}}$$

$$= \sqrt{\frac{(1.2-2.4)^2 + (3.4-2.4)^2 + (1.9-2.4)^2 + (3.2-2.4)^2}{4-1}}$$

$= 1.05357$

Similarly for dye $B$, $n_y = 4$, $\overline{Y} = 2.9$, $s_y = 0.26458$

The numbers of degrees of freedom is

$$v = \frac{\left[\frac{1.05357^2}{4} + \frac{0.26458^2}{4}\right]^2}{\frac{(1.05357^2/4)^2}{4-1} + \frac{(0.26458^2/4)^2}{4-1}} = 3\text{, rounded down to nearest integer.}$$

$$t_3 = \frac{(2.4 - 2.9 + 1)}{\sqrt{\frac{1.05357^2}{4} + \frac{0.26458^2}{4}}} = 0.921$$

The null and hypotheses are $H_0\colon \mu_X - \mu_Y \geq 1$ versus $H_1\colon \mu_X - \mu_Y < 1$.

Since, the alternate hypothesis is of the form $\mu_X - \mu_Y < \Delta$, the $P$-value is the area to the left of $t = 0.921$.

From the table given below, the $P$-value is greater than the threshold value of 0.05. Thus, we cannot conclude that the mean strength of crayons made with dye $B$ is greater than that made with dye $A$ .

Q9:

$\overline{X} = 66.5000$, $s_X = 19.460216$, $n_X = 6$,

$\overline{Y} = 28.0$, $s_Y = 17.776389$, $n_X = 5$.

Since the population standard deviations are assumed to be equal, estimate their common value with the pooled standard deviation

$$s_p = \sqrt{\frac{(n_X - 1)s_X^2 + (n_Y - 1)s_Y^2}{n_X + n_Y - 2}} = 18.730545.$$

The number of degrees of freedom is $\nu = n_X + n_Y - 2 = 6 + 5 - 2 = 9$.

$$t_9 = \frac{66.5000 - 28.0}{18.730545\sqrt{1/6 + 1/5}} = 3.394.$$

The null and alternate hypotheses are $H_0 : \mu_X - \mu_Y = 0$ versus $H_1 : \mu_X - \mu_Y \neq 0$.

Since the alternate hypothesis is of the form $\mu_X - \mu_Y \neq \Delta$, the $P$-value is the sum of the areas to the right of $t = 3.394$ and to the left of $t = -3.394$.

From the $t$ table, $0.005 < P < 0.01$. A computer package gives $P = 0.0079$.

We can conclude that the weights differ.

Q10:

The difference in etch rate between center and the edge for each observation are 6, −5, 2, 6, 2, 5, −3, 4, −3, and 6.

$\overline{D} = 2$,

$s_D = 4.2164$,

$n = 10$.

There are $10 - 1 = 9$ degrees of freedom.

The null and alternate hypotheses are $H_0$: $\mu_D = 0$ versus $H_1$: $\mu_D \neq 0$.

$$t = \frac{(\overline{D} - \mu_D)}{\left(\frac{s_D}{\sqrt{n}}\right)}$$

$$t = \frac{(2 - 0)}{\left(\frac{4.2164}{\sqrt{10}}\right)}$$

Since the alternate hypothesis is of the form $\mu_D \neq \Delta$, the $P$−value is the sum of the areas to the right of $t = 1.5$ and to the left of $t = -1.5$.

From the table given below, the $P$−value is greater than the threshold value of 0.05, hence we cannot conclude that the etch rates differ between the center and the edge.

Q11

There are $I = 4$ levels, with $J = 5$ observations at each level, for a total of $N = 20$ observations in all. To test at level $\alpha = 0.05$, we consult the Studentized Range table to find

$$q_{4,16,0.05} = 4.05.$$

The value of MSE is 27.48. Therefore

$$q_{I,N-I,\alpha}\sqrt{\text{MSE}/J} = 4.05\sqrt{27.48/5} = 9.49$$

The four sample means are as follows:

| Flux | A | B | C | D |
|---|---|---|---|---|
| Mean hardness | 251.8 | 259.4 | 267.8 | 269.2 |

There are three pairs whose difference is greater than 9.49.

## Q12 & Q13

Solution:

i. From the formulas, we need $\sum X^2$, $\sum \frac{T^2}{n}, \frac{G^2}{N}$

$$\sum X^2 = 170120$$

$$\sum \frac{T^2}{n} = \frac{1}{8}(743^2 + 657^2 + 604^2) = 168564.25$$

$$\frac{G^2}{N} = \frac{2004^2}{24} = 167334$$

$$SS_{total} = \sum X^2 - \frac{G^2}{N} = 2786$$

$$SS_{between} = \sum \frac{T^2}{n} - \frac{G^2}{N} = 1230.25$$

$$SS_{within} = \sum X^2 - \sum \frac{T^2}{n} = 1555.75$$

$$dof_{between} = \frac{24}{8} - 1 = 2$$

$$dof_{within} = 24 - \frac{24}{8} = 21$$

$$F = \frac{1230.25/2}{1555.75/21} = 8.303$$

ii. From the Sample T table, $df_1 = 2$, $df_2 = 21$, with $\alpha$=0.01, F value is 3.4668

Since the null hypothesis $H_0$ is the weight of the three populations are similar, i.e., air void level has no effect on the mean retained strength. Our computed F value exceeds the $F_{critical}$, we reject the null hypothesis at 0.01 level.

## Q14

Solution:

The type ii error is not rejecting the null hypothesis when it is false:

$$H_0: \mu \geq 3.2, H_a: \mu < 3.2$$

The probability of type ii error is not rejecting $H_0$ when it the actual thickness value is 3.1.

$$P(Do\ not\ reject\ H_0 | \mu = 3.1)$$

At $\alpha = 0.0005$, to not reject the $H_0$, a particular sample mean must result in $z > -z_{0.0005}$

$$-3.32 = \frac{\bar{X} - 3.2}{\frac{0.2}{\sqrt{50}}}$$

$$\bar{X} = 3.1061$$

$$P(Type\ II\ Error) = P\left(Z > \frac{3.1061 - 3.1}{\frac{0.2}{\sqrt{50}}}\right) = P(Z > 0.216) = P(Z < -0.216)$$

$$\approx P(Z < -0.22) = 0.4129$$

So the power of the hypothesis test is: 1-0.4129= 0.5871

## Q15

Solution:

The hypothesis is the sample average being larger than 1.27 M, and the alternative hypothesis is the sample average is being less than 1.27 M.

$$H_0: \mu \geq 1.27$$
$$H_1: \mu < 1.27$$

From the standard deviation tables, we know that $P(Z < -1.645) = 0.05$.

$$Z = \frac{\bar{X} - 1.27}{\frac{0.05}{\sqrt{20}}} < -1.645$$

So $\bar{X} < 1.2516$, so the mislabeled bottle's maximum Li+ molar concentration should be 1.282M

## Q16

Solution:

$$H_o: \mu_{day1} = \mu_{day2}$$
$$H_1: \mu_{day1} \neq \mu_{day2}$$

$$\bar{X}_{day1} = 75.33;\ \bar{X}_{day2} = 73.83$$
$$SS_{day1} = 8125.33;\ SS_{day2} = 8400.83$$
$$s_p^2 = \frac{8125.33 + 8400.83}{6 + 6 - 2} = 1652.62$$
$$SS_{\bar{X}_{day1} - \bar{X}_{day2}} = \sqrt{\frac{2 \times 1652.62}{6}} = 23.47$$
$$t = \frac{75.33 - 73.83 - 0}{23.47} = 0.0639$$

Look up t-table with dof = 6 + 6 - 2 = 10

$t_{crit} = \pm 2.228$

since $t < t_{crit}$, do not reject null

## Q17 & Q18

$$SS_D = 11.82 - \frac{(-2.8)^2}{4} = 9.86$$
$$S_D = \sqrt{\frac{9.86}{3}} = 1.81$$
$$S_{\bar{D}} = \frac{S_D}{\sqrt{4}} = 0.905$$

$$t = \frac{-2.8/4}{0.905} = -0.773$$

The interval is = -0.7 ± 3.182 x 0.905 and it’s from -3.579-2.180.