The lifetime of a transistor in a certain application has a lifetime that is random with probability density function

$$f(t) = \begin{cases} 0.2e^{-0.2t} & t > 0 \\ 0 & t \leq 0 \end{cases}$$

(a) Find the mean lifetime

$$\mu = \int_0^{\infty} 0.2te^{-0.2t}\,dt = -te^{-0.2t}\Big|_0^{\infty} - \int_o^{\infty} -e^{-0.2t}\,dt$$
$$= 0 - 5e^{-0.2t}\Big|_0^{\infty}$$
$$= 5$$

(b)Find the standard deviation of the lifetimes

$$\sigma^2 = \int_0^{\infty} 0.2t^2e^{-0.2t}\,dt - \mu^2 = -t^2e^{-0.2t}\Big|_0^{\infty} - \int_0^{\infty} -2te^{-0.2t}\,dt - 25$$
$$= 0 + 10\int_0^{\infty} 0.2te^{-0.2t}\,dt - 25$$
$$= 0 + 10(5) - 25$$
$$= 25$$
$$\sigma = 5$$

(c)Find the cumulative distribution of the lifetime

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

$If\ x \leq 0,\ F(x) = \int_{-\infty}^{x} 0dt = 0$

$If\ x > 0,\ F(x) = \int_{-\infty}^{0} 0dt + \int_0^{x} 0.2e^{-0.2t}\,dt = 1 - e^{-0.2x}$

(d)Find the probability that the life time will be less than 8 months

$$F(8) = \int_{-\infty}^{8} f(t)dt$$

$$F(8) = \int_{-\infty}^{0} 0dt + \int_0^{8} 0.2e^{-0.2t}\,dt = 1 - e^{-0.2\times 8} = 0.7981$$

The lifetime of a battery in a certain application is normally distributed with mean μ = 12 hours and standard deviation σ = 4 hours. Find the 12th percentile of the lifetimes.

Determine the 12th percentile of the lifetimes.

The z-score of the 12th percentile is ≈ -1.17

Given that $z = \frac{x-\mu}{\sigma}$

Therefore $-1.17 = \frac{x-12}{4}; x = 7.32$

A cylindrical hole is drilled in a block, and a cylindrical piston is placed in the hole. The clearance is equal to 0.5 the difference between the diameters of the hole and the piston. The diameter of the hole is normally distributed with mean 15.2 cm and standard deviation 0.025 cm, and the diameter of the piston is normally distributed with mean 14.77 cm and standard deviation 0.015 cm. Find the mean clearance.

$\mu_c = 0.5(\mu_{hole} - \mu_{piston}) = 0.5(15.2\ cm - 14.77\ cm) = 0.215\ cm$

A binary message m, where m is equal either to 0 or to 1, is sent over an information channel. Because of noise in the channel, the message received is X, where $X = m + E$, and E is a random variable representing the channel noise. Assume that if $X \le 0.5$ then the receiver concludes that $m = 0$ and that if $X > 0.5$ then the receiver concludes that $m = 1$. Assume that $E \sim N(0, 0.20)$.

a. If the true message is m = 0, what is the probability of an error, that is, what is the probability that the receiver concludes that m = 1?

If $m = 0$, then $X = E$, so $X \sim N(0, 0.20)$

$P(error) = P(X > 0.5)$. The z-score of 0.5 is $\frac{0.5-0}{\sqrt{0.20}} = 1.12$

The area to the right of $z = 1.12$ is 1-0.8686=0.1314

b. Let σ^2 denote the variance of E. What must be the value of σ^2 so that the probability of error when m = 0 is 0.03?

If $m = 0$, then $X = E$, so $X \sim N(0, \sigma^2)$

$P(error) = P(X > 0.5) = 0.03$. The z-score of 0.5 is 1.88

Therefore $\frac{0.5-0}{\sigma} = 1.88$. Solving for σ yelds $\sigma = 0.2660$, and $\sigma^2 = 0.0708$

A catalyst researcher states that the diameters, in microns, of the pores in a new product she has made have the exponential distribution with parameter $\lambda = 0.6$. What is the standard deviation of the pore diameters? Please report answers to two decimal places.

$$\sigma_x = \frac{1}{\lambda} = \frac{1}{0.6} = 1.67 \text{ microns}$$

A shoe manufacturer collected data regarding men's shoe sizes and women's shoe sizes. The distribution of men's shoe sizes is modeled as a normal random variable with a mean of $\mu = 11$ and a standard deviation of $\sigma = 2$. The women's shoe sizes are normally distributed with a mean of $\mu = 8$ and a standard deviation of $\sigma = 2$. What is the probability of finding a men's shoe with the size larger than 95% of women's shoes? Assume that the men's and women's sizes are equivalent, i.e. men's size 8 equals women's size 8. Please give your answer to two decimal places.

Solution:
First we need to calculate the women shoe size that is larger than 95% of of women's size:

$$P(X \le x) = 0.95 = P\left(Z < \frac{x-\mu_2}{\sigma_2}\right) = \Phi\left(\frac{x-\mu_2}{\sigma_2}\right) = \Phi\left(\frac{x-8}{2}\right)$$

$$\left(\frac{x-8}{2}\right) = 1.645, x = 11.29$$

Probability of men's shoe size larger than 11.3 is:

$$P(X > 11.3) = 1 - P(X \le 11.29) = 1 - P\left(Z < \frac{11.29-\mu_1}{\sigma_1}\right)$$

$$= 1 - \Phi\left(\frac{11.29-11}{2}\right) = 1 - \Phi(0.14) = 1 - 0.5557 = 0.44$$

A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 60 mile/hr and a standard deviation of 10 mile/hr. The probability of getting a speeding ticket is 30% if a car's speed is between 75 mile/hr and 85 mile/hr. The probability of getting a speeding ticket is 70% if a car's speed is above 85 mile/hr. What is the probability that a car picked at random will get a speeding ticket? Please give your answer to 4 decimal places.

Solution

The probability of finding a car with speed between 75 mile/hr and 85 mile/hr is:

$$
\begin{aligned}
P(75 < x < 85) &= P(x < 85) - P(x < 75) \\
&= P\left(\frac{W-\mu_1}{\sigma_1} < \frac{85-\mu_1}{\sigma_1}\right) - P\left(\frac{W-\mu_1}{\sigma_1} < \frac{75-\mu_1}{\sigma_1}\right) \\
&= \Phi\left(\frac{85-60}{10}\right) - \Phi\left(\frac{75-60}{10}\right) = \Phi(2.5) - \Phi(1.5) \\
&= 0.9938 - 0.9332 = 0.0606
\end{aligned}
$$

The probability of finding a car with speed above 85 mile/hr is:

$$P(x > 85) = 1 - P(x < 85) = 1 - 0.9938 = 0.0062$$

The probability of a car picked at random will get a speeding ticket is:

$$
\begin{aligned}
P(speeding\ ticket) &= P(75 < x < 85) * 0.3 + P(x > 85) * 0.7 \\
&= 0.0606 * 0.3 + 0.0062 * 0.7 = 0.02252 \cong 0.0225
\end{aligned}
$$

The time between arrivals of taxis at a busy intersection is exponentially distributed with a mean of 5 minutes in San Francisco. Determine x such that the probability that you wait more than x mins is 0.10. Please give your answer to two decimal places.

Solution:

Given that the average wait time is 5 mintues,

the rate $\lambda = \dfrac{1}{5 \text{ min}} = 0.2 \text{ min}^{-1}$

$$P(wait\ time > x) = 1 - F(x) = e^{-0.2x} = 0.10$$

$$x = 11.51 \text{ min}$$

A test instrument needs to be calibrated periodically to prevent measurement errors. After some time of use without calibration, it is known that the measurement error is given by the probability density function $f(x) = 1 - 0.5x$ for $0 < x < 2$ mm.

(a) If the measurement error within 0.5 mm is acceptable, what is the probability that the error is not acceptable before calibration?
(b) What is the probability that the measurement error is exactly 0.22 mm before calibration?

Solution:
(a)

$$P(0 < x < 0.5) = \int_0^{0.5} 1 - 0.5x\, dx = (0.5) - \frac{1}{4}(0.5)^2 = 0.4375$$

$$P(\mathit{not}\ \text{acceptable}) = 1 - P(0 < x < 0.5) = 0.5625$$

(b)
The answer is 0 because the question is asking for the probability of an exact value which will not be found in the given probability density function of a continuous random variable

Hits to a high-volume Web site are assumed to follow a Poisson distribution with a mean of 10,000 per day. Approximate the probability that the website has more than 20,200 hits in two days. Please give your answer to four decimal places.

Solution:

For two days, the mean and variance of the Poisson distribution is 20000.

P(X >20,200) ~ P(Z > (20200 – 20000) / sqrt(20000)) = P(Z>1.41) = 1 – P(Z<1.41) = 0.0793