Let X represent the number of tires with low air pressure on a randomly chosen car.

a. Which of the three functions below is a possible probability mass function of X?

	x				
	0	**1**	**2**	**3**	**4**
$p_1(x)$	**0.1**	**0.2**	**0.4**	**0.2**	**0.1**
$p_2(x)$	**0.1**	**0.1**	**0.2**	**0.2**	**0.2**
$p_3(x)$	**0.2**	**0.4**	**0.4**	**0.3**	**0.3**

b. For the possible probability mass function, compute μ_X and σ_X^2.

(a) $p_1(x)$ is the only probability mass function, because it is the only one whose probabilities sum to 1.

(b)

$$\begin{aligned} \mu_X &= 0(0.1) + 1(0.2) + 2(0.4) + 3(0.2) + 4(0.1) \\ &= 2.0, \\ \sigma_X^2 &= (0-2)^2(0.1) + (1-2)^2(0.2) + (2-2)^2(0.4) + (3-2)^2(0.2) + (4-2)^2(0.1) \\ &= 1.2 \end{aligned}$$

A chemical supply company ships a certain solvent in 10−gallon drums. Let X represent the number of drums ordered by a randomly chosen customer. Assume$X$ has the following probability mass function:

x	1	2	3	4	5
$p(x)$	0.32	0.15	0.15	0.03	0.35

Find the variance of the number of drums ordered.

Calculating the mean of X;

$$\mu_X = \sum_x xP(X = x)$$

$$\begin{aligned} \mu_X &= 1(0.32) + 2(0.15) + 3(0.15) + 4(0.03) + 5(0.35) \\ &= 2.9 \end{aligned}$$

Therefore, the variance of number of drums ordered:

$$\sigma_X^2 = \sum (x - \mu_X)^2 P(X = x)$$

$$\begin{aligned} \sigma_X^2 &= (1 - 2.9)^2(0.32) + (2 - 2.9)^2(0.15) + (3 - 2.9)^2(0.15) + (4 - 2.9)^2(0.03) + (5 - 2.9)^2(0.35) \\ &= 2.86 \end{aligned}$$

The four sides of a picture frame consist of two pieces selected from a population whose mean length is 30 cm with standard deviation 0.129 cm, and two pieces selected from a population whose mean length is 45 cm with standard deviation 0.38 cm. Assuming the four pieces are chosen independently, find the standard deviation of the perimeter.

Let $X1$ and $X2$ denote the lengths of the pieces chosen from the population with mean 30 and standard deviation 0.129, and let $Y1$ and $Y2$ denote the lengths of the pieces chosen from the population with mean 45 and standard deviation 0.38.

Therefore, the standard deviation of the perimeter;

$$\sigma_{X1+X2+Y1+Y2} = \sqrt{\sigma_{X1}^2 + \sigma_{X2}^2 + \sigma_{Y1}^2 + \sigma_{Y2}^2}$$

$$= \sqrt{0.129^2 + 0.129^2 + 0.38^2 + 0.38^2}$$

$= 0.568$

Of all the new vehicles of a certain model that are sold, 19% require repairs to be done under warranty during the first year of service. A particular dealership sells 20 such vehicles. What is the probability that fewer than 3 of them require warranty repairs?
Carry four significant digits throughout your calculations.

Let X be the number of vehicles that require warranty repairs. Then $X \sim \text{Bin}(20, 0.19)$

For: $P\ (X < 3)$

$$= \frac{20!}{0!(20-0)!}(0.19)^0(1-0.19)^{20-0} + \frac{20!}{1!(20-1)}(0.19)^1(1-0.19)^{20-1} +$$

$$\frac{20!}{2!(20-2)!}(0.19)^2(1-0.19)^{20-2}$$

$= 0.2386$

Of the items manufactured by a certain process, 20% are defective. Of the defective items, 60% can be repaired.
a. Find the probability that a randomly chosen item is defective and cannot be repaired.
b. Find the probability that exactly 3 of 15 randomly chosen items are defective and cannot be repaired. Carry three significant digits throughout your calculations.

(a) Let D denote the event that the item is defective and let R denote the event that the item can be repaired. Then $P(D) = 0.2$ and $P(R|D) = 0.6$. Then

$$\begin{aligned} P(D \cap R^c) &= P(R^c|D)P(D) \\ &= [1 - P(R|D)]P(D) \\ &= (1 - 0.6)(0.2) \\ &= 0.08. \end{aligned}$$

(b) Let X be the number of items out of 15 that are defective and cannot be repaired. Then $X \sim \text{Bin}(15, 0.08)$.

$$\begin{aligned} P(X = 3) &= \frac{15!}{3!(15-3)!}(0.08)^3(1-0.08)^{(15-3)} \\ &= 0.0857 \end{aligned}$$

The number of messages received by a computer bulletin board is a Poisson random variable with a mean rate of 7 messages per hour. What is the probability that 4 messages are received in a given hour?

Let X be the number of messages received in one hour.

Since, the mean rate is 7 messages per hour, $X \sim$ Poisson(7).

$$P(X = x) = e^{-\lambda}\left(\frac{\lambda^x}{x!}\right)$$

$$P(X = 4) = e^{-7}\left(\frac{7^4}{4!}\right)$$

$= 0.0912$

The number of messages received by a computer bulletin board is a Poisson random variable with a mean rate of 9 messages per hour. What is the probability that fewer than three messages are received in 0.29 hour?

Let X be the number of messages received in 0.29 hour. Since, the mean rate is 9 messages per hour, $X \sim \text{Poisson}(2.61)$.

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$e^{-2.61}\frac{(2.61)^0}{0!} + e^{-2.61}\frac{(2.61)^1}{1!} + e^{-2.61}\frac{(2.61)^2}{2!}$$

$$= 0.073535 + 0.191925 + 0.250462$$

$$= 0.515922$$

The probability that a computer running a certain operating system crashes on any given day is 0.14. Find the probability that the computer crashes for the first time on the 17th day after the operating system is installed.

Let X be the number of the day on which the computer crashes.

Then, $X \sim \text{Geom}(0.14)$

$P(X = x) = p(1 - p)^{x - 1}$

$P(X = 17) = (0.14)(1 - 0.14)^{17 - 1}$

$= 0.0125$