Problem Set 1 – Solution

NANO 114 Probability and Statistical Methods for Engineers

Q1:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.95 + 0.9 - 0.88 = 0.97$$

Q2:

$$P(match) = P(BB) + P(WW) = \frac{8}{14} \times \frac{4}{6} + \frac{6}{14} \times \frac{2}{6} = \frac{44}{84} = 0.5238$$

Q3:

Let A represent the event that the biotechnology company is profitable, and let B represent the event that the information technology company is profitable. Then P(A) = 0.2, and P(B)=0.15.

a) $P(A \cap B) = P(A)P(B) = 0.2 \times 0.15 = 0.03$
b) $P(A^c \cap B^c) = P(A^c)P(B^c) = (1 - 0.2)(1 - 0.15) = 0.68$
c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B) = 0.2 + 0.15 - (0.2)(0.15) = 0.32$

Q4:

Let E denote the event that a parcel is sent express (so E^c denotes the event that a parcel is sent standard), and let N denote the event that a parcel arrives the next day. Then P(E) = 0.25, P(N|E) = 0.95, and P(N|E^c) =0.8).

a) $P(E \cap N) = P(E)P(N|E) = 0.25 \times 0.95 = 0.2375$
b) $P(N) = P(N|E)P(E) + P(N|E^c)P(E^c) = 0.95 \times 0.25 + 0.8 \times (1 - 0.25) = 0.8375$
c) $P(E|N) = \frac{P(N|E)P(E)}{P(N|E)P(E) + P(N|E^c)P(E^c)} = \frac{0.95 \times 0.25}{0.95 \times 0.25 + 0.8 \times (1 - 0.25)} = 0.2836$

Q5:

$P(system\ functions) = P[(A \cap B) \cup (C \cup D)]$

Now $P(A \cap B) = P(A)P(B) = (1 - 0.1)(1 - 0.05) = 0.855$ and $P(C \cup D) = P(C) + P(D) - P(C \cap D) = (1 - 0.1) + (1 - 0.2) - (1 - 0.1)(1 - 0.2) = 0.98$.

Therefore

$$\begin{aligned} P[(A \cap B) \cup (C \cup D)] &= P(A \cap B) + P(C \cup D) - P[(A \cap B) \cap (C \cup D)] \\ &= P(A \cap B) + P(C \cup D) - P(A \cap B)P(C \cup D) \\ &= 0.855 + 0.98 - (0.855)(0.98) = 0.9971 \end{aligned}$$

Q6:
Machines X and Y produce 35% and 65% of the total production of a certain drug in a pharmaceutical company respectively. The probability that machine X produces an impure drug is 0.02. If the probability that a pure drug taken at random from the production line is 94.75%, what is the probability that machine Y produces an impure drug? Please provide your answer to 2 decimal places.

Solution:

$A = pure\ drug$
$B = impure\ drug$

$$P(B) = 1 - P(A) = 1 - 0.9475 = 0.0525$$

$$\begin{aligned} P(B) &= P(B\ \&\ machine\ X) + P(B\ \&\ machine\ Y) \\ &= P(B|machine\ X) \cdot P(machine\ X) + P(B|machine\ Y) \cdot P(machine\ Y) \\ &= 0.02 \cdot 0.35 + P(B|machine\ Y) \cdot 0.65 \end{aligned}$$

$$0.0525 = 0.02 \cdot 0.35 + P(B|machine\ Y) \cdot 0.65$$

$$P(B|machine\ Y) = 0.07$$

Q7:

A batch of manufacturing parts contains 2 defective parts and 9 non-defective parts. A sample of 5 parts is selected without replacement. What is the probability of the sample containing 1 defective part? Please provide your answer to 2 decimal places.

Solution:
The number of ways to select 5 parts from the batch is:

$${}^{11}C_5 = \frac{11!}{5!6!} = 462$$

The number of ways to select 1 defective part from the batch is:

$${}^{2}C_1 = \frac{2!}{1!1!} = 2$$

The number of ways to select 4 non-defective parts from the batch is:

$${}^{9}C_4 = \frac{9!}{4!5!} = 126$$

Then the probability of choose a sample containing 1 defective part is:

$$P = \frac{{}^{2}C_1\,{}^{9}C_4}{{}^{11}C_5} = \frac{2 \times 126}{462} = 0.55$$

Answer: 0.55

Q8:

The manufacturer of a chemical sources a particular reagent from companies A, B and C in the ratio of 20%, 30% and 50% respectively. Because the companies have different quality control, the probabilities that a reagent sample from companies A, B and C meets the required purity standard of > 99.99% is 80%, 90% and 95% respectively. You pick up a random sample and tested it, and it did not meet the required purity standard. What is the probability that the sample came from company A? Please provide your answer to three decimal places.

Solution: Let P(A), P(B), P(C) be the probability of reagent from companies A, B and C respectively. P(A) = 0.2, P(B) = 0.3, P(C) = 0.5. Let F be the event that a reagent does not meet the required purity. Conditional probability, P(F|A) = 0.2, P(F|B) = 0.1 and P(F|C) = 0.05.

Hence based on Bayer's Rule:

$$P(A|F) = \frac{P(A)\times P(F|A)}{P(A)\times P(F|A) + P(B)\times P(F|B) + P(C)\times P(F|C)}$$
$$= \frac{0.2\cdot 0.2}{0.2\cdot 0.2 + 0.3\cdot 0.1 + 0.5\cdot 0.05} \cong 0.421$$

Answer: 0.421

Q9:

We have a group of 30 students. What is the probability that at least two students have the same birthday? You can ignore leap year considerations, and give your answers to 3 decimal places.

Let P(A) be the probability of at least two students have the same birthday. Considering the complement event, all 30 students have different birthdays.

Total possible combinations of birthdays = 365^{30} (there are 365 choices for each birthday)
Total number of combinations where all birthdays are unique = $365\times 364\times 363\times \cdots\times 336 = \frac{365!}{335!}$

$$P(A^c) = \frac{365!/335!}{365^{30}} = 0.294$$

$$1 - P(A^c) = 0.706$$

Answer: 0.706

Q10 & Q11:

A production plant produces steel cans. 10 cans are taken for examination; 3 fail inspection but the other 7 pass.

(i.) If 3 cans are selected at random what is the probability that one fails and the other two pass? Please provide your answer to three decimal places.

(ii.) If 3 cans are selected at random what is the probability that the first two pass and the last one fails? Please provide your answer to three decimal places.

Solutions

(i.) $$\frac{{}^{3}C_{1}\times{}^{7}C_{2}}{{}^{10}C_{3}}=\frac{63}{120}=0.525$$

For this problem order does not matter so we will use combinations. ${}^{10}C_3$ represents all the possible ways to choose 3 cans from 10. In the numerator we are looking for ways to choose 2 passing cans out of 7 (${}^{7}C_2$) and 1 failing can out of 3 (${}^{3}C_1$).

(ii.) $$\frac{{}^{7}C_{2}}{{}^{10}C_{2}}\times\frac{{}^{3}C_{1}}{{}^{8}C_{1}}=\frac{21}{45}\times\frac{3}{8}=0.175$$

For this problem order matters, however, we are taking care of this by splitting the solution up into two terms. We first want to look at the first two cans picked (${}^{10}C_2$). For these we want 2 passing cans out of the 7 (${}^{7}C_2$). Next, after those two have already been taken we look at the third can picked. Now there are only 8 cans left to pick from (${}^{8}C_1$). For this we want 1 failed can out of the 3 (${}^{3}C_1$).

Note: You can also get this answer by dividing the answer from part (i.) by 3. There are three possible combinations that can produce the desired outcome for part (i.): PPF, PFP, FPP. However, only one of these combinations, PPF, qualifies for the stipulations set in part (ii.). Since each of the three combinations has equal probability of occurring the value from part (i.) can simply be divided by 3.

$$\frac{0.525}{3}=0.175$$