CENG/NANO 114

Probability and Statistical Methods for Engineers

Problem Set 2

Note: Questions denoted with a * are meant to be more difficult than usual. Hints are provided for a selected number of questions.

1. A discrete probability random variable can take integer values from 1 to 5 inclusive, i.e., {1, 2, 3, 4, 5}. The probability of the random variable taking on a particular value x is inversely proportion to that value, i.e., k / x.
   i. Determine the value of k that makes this a valid probability distribution.
   ii. Calculate the mean and variance of the random variable. (Hint: The mean is greater than 2.)

2. A joint probability mass function (PMF) of two random variables X and Y is given by the table below. Each cell represents the probability P(X=x, Y=y).

		X		
		1	**2**	**3**
y	**1**	0	0.05	0.3
	2	0.2	0.1	0
	3	0.02	0.3	0.03

   i. Determine the marginal PMFs for X and Y.
   ii. Determine the PMF of the random variable given by Z = XY.
   iii. What is the probability of X is odd given that Y = 3? (Hint: It's less than 0.5)
   iv. Calculate E[Y|X=2].
   v. Are X and Y independent?(Hint:You can use properties of independent random variables to verify.)

3. There are 200 fourth-year undergraduate students in a department. They have a core course need to learn this quarter. If you are one of them, what is the probability that exactly one other student has the same birthday as you regardless of born year? Give the exact probability and approximate it by using the Poisson PMF. (You can exclude the birthday on Feb 29.)(Hint: Exact probability and Poisson PMF are two different types. Under some situation, you can use Poisson PMF as an approximating method.)

4. Let X denote the number of flaws on the surface of a randomly selected boiler of a certain type. Suppose X has a Poisson distribution with λ=5.
   a. Write down the probability of a randomly selected boiler has exactly k flaws.
   b. What is the probability that a randomly selected boiler contains at most two flaws? (Hint: it is greater than 0.1)

c. What is the probability that a randomly selected boiler has more than three flaws given that it has more than one flaw?

5. The PMF for X, the random variable of the number of major defects on a randomly selected appliance of a certain type is given as:

X	0	1	2	3	4
P(x)	0.04	0.17	0.46	0.27	0.06

Compute the following:
   i. E(X)
   ii. Var(X)
   iii. The standard deviation of X.

6. The probability of a successful optical alignment in the assembly of an optical data storage product is 0.8. Attempts are made until a successful alignment is obtained. Assume that the trials for alignment are independent.(Hint: Identify the situation of exactly, at most, at least four tries. Some of them may be got directly and for others the complement set may be taken into account to make it simpler.)
   i. What is the probability that a successful alignment requires exactly four tries?
   ii. What is the probability that a successful alignment requires at most four tries?
   iii. What is the probability that a successful alignment requires at least four tries?

7. Assume that the wavelengths of photo-synthetically active radiations (PAR) are uniformly distributed at integer nanometers in the red spectrum from 675 to 700 nm, inclusive.
   i. What are the mean and variance of the wavelength distribution for this radiation?
   ii. If the wavelengths are uniformly distributed at integer nanometers from 175 to 200 nanometers inclusive, how do the mean and variance of the wavelength distribution compare to the previous part? Explain.

8. Consider the time to recharge the flash in cell-phone cameras. Assume that the probability that a camera passes the test is 0.75 and the cameras perform independently. Determine the following:
   i. Probability that the second failure occurs on the tenth camera tested.
   ii. Probability that the second failure occurs in tests of four or fewer cameras.
   iii. Expected number of cameras tested to obtain the third failure.
   iv. If we test 100 cameras, calculate the mean and standard deviation of cameras passing the test.

9. The number of cracks in a section of interstate highway that are significant enough to require repair is assumed to follow a Poisson distribution with a mean of 2 cracks per mile.
   i. What is the probability that there are no cracks that require repair in 6 miles of highway? (Hint: It is a very small number.)
   ii. What is the probability that at least one crack requires repair in 1.5 miles of highway?
   iii. If the number of cracks is related to the vehicle load on the highway and some

sections of the highway have a heavy load of vehicles whereas other sections carry a light load, what do you think about the assumption of a Poisson distribution for the number of cracks that require repair?

*10. In many studies of random process (e.g., diffusion, stock markets, etc.), a random walk model is used. In the simplest case, a 1D random walk comprises of a sequence of N independent equal-length steps, either forward with probability p or backwards with probability $(1-p)$. Derive an expression for the expectation of the displacement, assuming each step is of length d. Note that when p = 0.5, this is an unbiased random walk and the expectation is of course 0. But a general solution is sought.